EXHIBIT 1022

Bernoulli's principle

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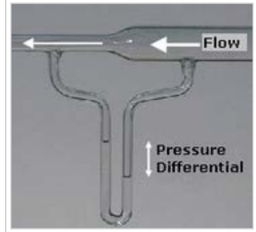

A flow of air into a venturi meter. The kinetic energy increases at the expense of the fluid pressure, as shown by the difference in height of the two columns of water.

In fluid dynamics, **Bernoulli's principle** states that for an inviscid flow, an increase in the speed of the fluid occurs simultaneously with a decrease in pressure or a decrease in the fluid's potential energy.[1][2] Bernoulli's principle is named after the Dutch–Swiss mathematician Daniel Bernoulli who published his principle in his book *Hydrodynamica* in 1738.[3]

Bernoulli's principle can be applied to various types of fluid flow, resulting in what is loosely denoted as **Bernoulli's equation**. In fact, there are different forms of the Bernoulli equation for different types of flow. The simple form of Bernoulli's principle is valid for incompressible flows (e.g. most liquid flows) and also for compressible flows (e.g. gases) moving at low Mach numbers. More advanced forms may in some cases be applied to compressible flows at higher Mach numbers (see the derivations of the Bernoulli equation).

Bernoulli's principle is equivalent to the principle of conservation of energy. This states that in a steady flow the sum of all forms of mechanical energy in a fluid along a streamline is the same at all points on that streamline. This requires that the sum of kinetic energy and potential energy remain constant. If the fluid is flowing out of a reservoir the sum of all forms of energy is the same on all streamlines because in a reservoir the energy per unit mass (the sum of pressure and gravitational potential ρgh) is the same everywhere.[4]

Fluid particles are subject only to pressure and their own weight. If a fluid is flowing horizontally and along a section of a streamline, where the speed increases it can only be because the fluid on that section has moved from a region of higher pressure to a region of lower pressure; and if its speed decreases, it can only be because it has moved from a region of lower pressure to a region of higher pressure. Consequently, within a fluid flowing horizontally, the highest speed occurs where the pressure is lowest, and the lowest speed occurs where the pressure is highest.

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Incompressible flow equation

In most flows of liquids, and of gases at low Mach number, the mass density of a fluid parcel can be considered to be constant, regardless of pressure variations in the flow. For this reason the fluid in such flows can be considered to be incompressible and these flows can be described as incompressible flow. Bernoulli performed his experiments on liquids and his equation in its original form is valid only for incompressible flow.

The original form of Bernoulli's equation[5] is:

$$\frac{v^2}{2} + gz + \frac{p}{\rho} = \text{constant}$$

where:

v is the fluid flow speed at a point on a streamline,
g is the acceleration due to gravity,
z is the elevation of the point above a reference plane, with the positive z-direction in the direction opposite to the gravitational acceleration,
p is the pressure at the point, and
ρ is the density of the fluid at all points in the fluid.

The following assumptions must be met for the equation to apply:

- The fluid must be incompressible - even though pressure varies, the density must remain constant.
- The streamline must not enter a boundary layer. (Bernoulli's equation is not applicable where there are viscous forces, such as in a boundary layer.)

By multiplying by ρ, the above equation can be rewritten as:

$$\frac{1}{2}\rho v^2 + \rho g z + p = \text{constant}$$

or:

$$q + \rho g h = p_0 + \rho g z = \text{constant}$$

where:

$q = \frac{1}{2}\rho v^2$ is dynamic pressure,

$h = z + \frac{p}{\rho g}$ is the piezometric head or hydraulic head (the sum of the elevation *z* and the pressure head)[6][7] and

$p_0 = p + q$ is the **total pressure** (the sum of the static pressure *p* and dynamic pressure *q*).[8]

The constant in the Bernoulli equation can be normalised. A common approach is in terms of **total head** or **energy head** *H*:

$H = z + \frac{p}{\rho g} + \frac{v^2}{2g} = h + \frac{v^2}{2g}$, so divide the above constant by ρ and g to get the total head *H* in terms of metres of fluid column.[7][6]

The above equations suggest there is a flow speed at which pressure is zero and at higher speeds the pressure is negative. Gases and liquids are not capable of negative absolute pressure, or even zero pressure, so clearly Bernoulli's equation ceases to be valid before zero pressure is reached. The above equations use a linear relationship between flow speed squared and pressure. At higher velocities in liquids, non-linear processes such as (viscous) turbulent flow and cavitation occur. At higher flow speeds in gases the changes in pressure become significant so that the assumption of constant density is invalid.

Simplified form

In several applications of Bernoulli's equation, the change in the $\rho g z$ term along streamlines is zero or so small it can be ignored: for instance in the case of airfoils at low Mach number. This allows the above equation to be presented in the following simplified form:

$$p + q = p_0$$

where p_0 is called total pressure, and q is dynamic pressure[9]. Many authors refer to the pressure p as static pressure to distinguish it from total pressure p_0 and dynamic pressure q. In *Aerodynamics*, L.J. Clancy writes: "To distinguish it from the total and dynamic pressures, the actual pressure of the fluid, which is associated not with its motion but with its state, is often referred to as the static pressure, but where the term pressure alone is used it refers to this static pressure."[10]

The simplified form of Bernoulli's equation can be summarized in the following memorable word equation:

static pressure + dynamic pressure = total pressure[10]

Every point in a steadily flowing fluid, regardless of the fluid speed at that point, has its own unique static pressure *p*, dynamic pressure *q*, and total pressure p_0.

The significance of Bernoulli's principle can now be summarized as "total pressure is constant along a streamline." Furthermore, if the fluid flow originated in a reservoir, the total pressure on every streamline is the same and Bernoulli's principle can be summarized as "total pressure is constant everywhere in the fluid flow." However, it is important to remember that Bernoulli's principle does not apply in the boundary layer.

Applicability of incompressible flow equation to flow of gases

Bernoulli's equation is sometimes valid for the flow of gases provided that there is no transfer of kinetic or potential energy from the gas flow to the compression or expansion of the gas. If both the gas pressure and volume change simultaneously, then work will be done on or by the gas. In this case, Bernoulli's equation can not be assumed to be valid. However if the gas process is entirely isobaric, or isochoric, then no work is done on or by the gas, (so the simple energy balance is not upset). According to the gas law, an isobaric or isochoric process is ordinarily the only way to ensure constant density in a gas. Also the gas density will be proportional to the ratio of pressure and absolute temperature, however this ratio will vary upon compression or expansion, no matter what non-zero quantity of heat is added or removed. The only exception is if the net heat transfer is zero, as in a complete thermodynamic cycle, or in an individual isentropic (frictionless adiabatic) process, and even then this reversible process must be reversed, to restore the gas to the original pressure and specific volume, and thus density. Only then is the original, unmodified Bernoulli equation applicable. In this case the equation can be used if the flow speed of the gas is sufficiently below the speed of sound, such that the variation in density of the gas (due to this effect) along each streamline can be ignored. Adiabatic flow at less than Mach 0.3 is generally considered to be slow enough.

Unsteady potential flow

The Bernoulli equation for unsteady potential flow is used in the theory of ocean surface waves and acoustics.

For an irrotational flow, the flow velocity can be described as the gradient $\nabla\varphi$ of a velocity potential φ. In that case, and for a constant density ρ, the momentum equations of the Euler equations can be integrated to:[11]

$$\frac{\partial\varphi}{\partial t} + \frac{1}{2}v^2 + \frac{p}{\rho} + gz = f(t),$$

which is a Bernoulli equation valid also for unsteady — or time dependent — flows. Here $\partial\varphi/\partial t$ denotes the partial derivative of the velocity potential φ with respect to time *t*, and $v = |\nabla\varphi|$ is the flow speed. The function *f(t)* depends only on time and not on position in the fluid. As a result, the Bernoulli equation at some moment *t* does not only apply along a certain streamline, but in the whole fluid domain. This is also true for the special case of a steady irrotational flow, in which case *f* is a constant.[11]

Further *f(t)* can be made equal to zero by incorporating it into the velocity potential using the transformation

$$\Phi = \varphi - \int_{t_0}^{t} f(\tau)\,\mathrm{d}\tau \quad \text{resulting in} \quad \frac{\partial\Phi}{\partial t} + \frac{1}{2}v^2 + \frac{p}{\rho} + gz = 0.$$

Note that the relation of the potential to the flow velocity is unaffected by this transformation: $\nabla\Phi = \nabla\varphi$.

The Bernoulli equation for unsteady potential flow also appears to play a central role in Luke's variational principle, a variational description of free-surface flows using the Lagrangian (not to be confused with Lagrangian coordinates).

Compressible flow equation

Bernoulli developed his principle from his observations on liquids, and his equation is applicable only to incompressible fluids, and compressible fluids at very low speeds (perhaps up to 1/3 of the sound speed in the fluid). It is possible to use the fundamental principles of physics to develop similar equations applicable to compressible fluids. There are numerous equations, each tailored for a particular application, but all are analogous to Bernoulli's equation and all rely on nothing more than the fundamental principles of physics such as Newton's laws of motion or the first law of thermodynamics.

Compressible flow in fluid dynamics

A useful form of the equation, suitable for use in compressible fluid dynamics, is:

$$\frac{v^2}{2} + gz + \left(\frac{\gamma}{\gamma - 1}\right)\frac{p}{\rho} = \text{constant}$$ [12] (constant along a streamline)

where:

γ is the ratio of the specific heats of the fluid
p is the pressure at a point
ρ is the density at the point
v is the speed of the fluid at the point
g is the acceleration due to gravity
z is the elevation of the point above a reference plane

In many applications of compressible flow, changes in elevation are negligible compared to the other terms, so the term gz can be omitted. A very useful form of the equation is then:

$$\frac{v^2}{2} + \left(\frac{\gamma}{\gamma - 1}\right)\frac{p}{\rho} = \left(\frac{\gamma}{\gamma - 1}\right)\frac{p_0}{\rho_0}$$

where:

p_0 is the total pressure
ρ_0 is the total density

Compressible flow in thermodynamics

Another useful form of the equation, suitable for use in thermodynamics, is:

$$\frac{v^2}{2} + gz + w = \text{constant}$$ [13]

w is the enthalpy per unit mass, which is also often written as h (not to be confused with "head" or "height").

Note that $w = \epsilon + \frac{p}{\rho}$ where ϵ is the thermodynamic energy per unit mass, also known as the specific internal energy or "sie."

The constant on the right hand side is often called the Bernoulli constant and denoted b. For steady inviscid adiabatic flow with no additional sources or sinks of energy, b is constant along any given streamline. More generally, when b may vary along streamlines, it still proves a useful parameter, related to the "head" of the fluid (see below).

When the change in gz can be ignored, a very useful form of this equation is:

$$\frac{v^2}{2} + w = w_0$$

where w_0 is total enthalpy.

When shock waves are present, in a reference frame moving with a shock, many of the parameters in the Bernoulli equation suffer abrupt changes in passing through the shock. The Bernoulli parameter itself, however, remains unaffected. An exception to this rule is radiative shocks, which violate the assumptions leading to the Bernoulli equation, namely the lack of additional sinks or sources of energy.

Derivations of Bernoulli equation

Bernoulli equation for incompressible fluids	[show]

Bernoulli equation for compressible fluids	[show]

Real world application

In every-day life there are many observations that can be successfully explained by application of Bernoulli's principle.

- The relative air flow parallel to the top surface of an aircraft wing or helicopter rotor blade is faster than along the bottom surface. Bernoulli's principle states that the pressure on the surfaces of the wing or rotor blade will be lower above than below, and this pressure difference results in an upwards lift force.[14][15] If the relative air flows across the top and bottom surfaces of a wing or rotor are known, then lift forces can be calculated (to a good approximation) using Bernoulli's equations — established by Bernoulli over a century before the first man-made wings were used for the purpose of flight. Note that Bernoulli's principle does not explain *why* the

air flows faster past the top of the wing and slower past the under-side. To understand *why*, it is helpful to understand circulation, the Kutta condition and the Kutta–Joukowski theorem.
Besides, Newton's third law states that forces only exist in pairs, so the air's upwards force on the wing coexists with the wing's downward force on the air, which results in a downward acceleration of air.

- The carburetor used in many reciprocating engines contains a venturi to create a region of low pressure to draw fuel into the carburetor and mix it thoroughly with the incoming air. The low pressure in the throat of a venturi can be explained by Bernoulli's principle - in the narrow throat, the air is moving at its fastest speed and therefore it is at its lowest pressure.
- The pitot tube and static port on an aircraft are used to determine the airspeed of the aircraft. These two devices are connected to the airspeed indicator which determines the dynamic pressure of the airflow past the aircraft. Dynamic pressure is the difference between stagnation pressure and static pressure. Bernoulli's principle is used to calibrate the airspeed indicator so that it displays the indicated airspeed appropriate to the dynamic pressure.[16]
- The flow speed of a fluid can be measured using a device such as a Venturi meter or an orifice plate, which can be placed into a pipeline to reduce the diameter of the flow. For a horizontal device, the continuity equation shows that for an incompressible fluid, the reduction in diameter will cause an increase in the fluid flow speed. Subsequently Bernoulli's principle then shows that there must be a decrease in the pressure in the reduced diameter region. This phenomenon is known as the Venturi effect.
- The maximum possible drain rate for a tank with a hole or tap at the base can be calculated directly from Bernoulli's equation, and is found to be proportional to the square root of the height of the fluid in the tank. This is Torricelli's law, showing that Torricelli's law is compatible with Bernoulli's principle. Viscosity lowers this drain rate. This is reflected in the discharge coefficient which is a function of the Reynold's number and the shape of the orifice.[17]
- The principle also makes it possible for sail-powered craft to travel faster than the wind that propels them (if friction can be sufficiently reduced). If the wind passing in front of the sail is fast enough to experience a significant reduction in pressure, the sail is pulled forward, in addition to being pushed from behind. While boats in water must contend with the friction of the water along the hull, ice sailing and land sailing vehicles can travel faster than the wind.[18]

Misunderstandings about the generation of lift

Many explanations for the generation of lift can be found; but some of these explanations can be misleading, and some are false. This has been a source of heated discussion over the years. In particular, there has been debate about whether lift is best explained by Bernoulli's principle or Newton's Laws. Modern writings agree that Bernoulli's principle and Newton's Laws are both relevant and correct.[19][20]

Several of these explanations use Bernoulli's principle to connect the flow kinematics to the flow-induced pressures. In case of incorrect (or partially correct) explanations of lift, also relying at some stage on Bernoulli's principle, the errors generally occur in the assumptions on the flow kinematics, and how these are produced. It is not Bernoulli's principle itself that is questioned because this principle is well established[21][22][23].

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Notes

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8. ^ Oertel, Herbert; Prandtl, Ludwig; Böhle, M.; Mayes, Katherine (2004), *Prandtl's Essentials of Fluid Mechanics*, Springer, ISBN 0387404376, 723 pages. See pp. 70–71.
9. ^ NASA's guide to Bernoulli's Equation (https://web.archive.org/web/20081231133251/http://www.grc.nasa.gov/WWW/K-12/airplane/bern.html)
10. ^ ***a b*** Clancy, L.J., *Aerodynamics*, Section 3.5.
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13. ^ Van Wylen, G.J., and Sonntag, R.E., (1965), *Fundamentals of Classical Thermodynamics*, Section 5.9, John Wiley and Sons Inc., New York
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16. ^ Clancy, L.J., *Aerodynamics*, section 3.8
17. ^ Mechanical Engineering Reference Manual Ninth Edition
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See also

- Terminology in fluid dynamics
- Navier–Stokes equations – for the flow of a viscous fluid
- Euler equations – for the flow of an inviscid fluid
- Hydraulics – applied fluid mechanics for liquids
- Iomega Bernoulli Box – a disk storage system utilizing Bernoulli's principle.

External links

- Denver University - Bernoulli's equation and Pressure measurement (https://web.archive.org/web/20081231133251/http://mysite.du.edu/~jcalvert/tech/fluids/bernoul.htm)
- Millersville University - Applications of Euler's Equation (https://web.archive.org/web/20081231133251/http://www.millersville.edu/~jdooley/macro/macrohyp/eulerap/eulap.htm)
- Nasa - Beginner's Guide to Aerodynamics (https://web.archive.org/web/20081231133251/http://www.grc.nasa.gov/WWW/K-12/airplane/bga.html)